Physics of Machines: RMF, Flux Paths, EM Force

Abstract

Rotating fields, flux paths, and electromagnetic force explained from first principles, with 6 solved problems (beginner to advanced) and LaTeX-ready formulas.

1 Why these three ideas matter

Electrical machines (motors, generators, transformers) rest on three pillars:

1. Rotating magnetic field (RMF)
2. Flux path and reluctance
3. Electromagnetic force/torque

2 Rotating magnetic field (RMF)

In a balanced three-phase stator, the superposition of phase mmfs yields a constant-magnitude field rotating at synchronous speed

$$n_s = \frac{120f}{p} \quad \text{(rpm)},$$

with frequency f and pole count p. The electrical angular frequency is

$$\omega_s = \frac{2\pi n_s}{60}.$$

Induction motors operate with slip

$$s = \frac{n_s - n_r}{n_s}.$$

3 Flux path and magnetic circuits

Using the magnetic-circuit analogy:

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}_m}, \qquad \mathcal{F} = NI, \qquad \mathcal{R}_m = \frac{\ell}{\mu_0 \mu_r A}.$$

For transformers,

$$v(t) = N\frac{d\Phi}{dt}, \qquad V_{\text{rms}} \approx 4.44 f N \Phi_{\max}.$$

4 Electromagnetic force and torque

Lorentz force:

$$\mathbf{F} = I\,\ell \times \mathbf{B}.$$

Torque via co-energy:

$$T(\theta, i) = \frac{\partial W_{\mathrm{co}}(\theta, i)}{\partial \theta}, \qquad W_{\mathrm{co}} = \frac{1}{2} L(\theta) i^2.$$

If $L(\theta) = L_0 + L_1 \cos(m\theta)$,

$$T(\theta) = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}.$$

For round-rotor synchronous machines (neglecting resistance):

$$P_e = \frac{3VE}{X_s} \sin\delta, \qquad T_e = \frac{P_e}{\omega_m}.$$

5 Worked problems

Beginner

B1. Synchronous speed Given $p = 4$ and $f = 60\,\mathrm{Hz}$,

$$n_s = \frac{120f}{p} = \frac{120 \times 60}{4} = 1800\,\mathrm{rpm}.$$

B2. Flux in a core $N = 500, I = 0.2\,\mathrm{A}, \ell = 0.3\,\mathrm{m}, A = 2.0 \times 10^{-4}\,\mathrm{m}^2, \mu_r = 4000.$

$$\mathcal{R}_m = \frac{0.3}{(4\pi \times 10^{-7}) \times 4000 \times 2.0 \times 10^{-4}} \approx 2.985 \times 10^5 \text{ A-turn/Wb}.$$

$$\Phi = \frac{NI}{\mathcal{R}_m} = \frac{100}{2.985 \times 10^5} \approx 3.35 \times 10^{-4}\,\mathrm{Wb}.$$

Intermediate

I1. Slip and rotor frequency 4-pole, $f = 60\,\mathrm{Hz}$, $n_r = 1740\,\mathrm{rpm}$.

$$n_s = 1800 \text{ rpm}, \quad s = \frac{60}{1800} = 0.0333, \quad f_r = sf \approx 2\,\mathrm{Hz}.$$

I2. Lorentz force $B = 0.8\,\mathrm{T}$, $I = 25\,\mathrm{A}$, $\ell = 0.15\,\mathrm{m}$:

$$F = BI\ell = 0.8 \times 25 \times 0.15 = 3.0 \text{ N}.$$

Advanced

A1. Torque from $L(\theta)$ $L(\theta) = L_0 + L_1 \cos(2\theta)$, $L_0 = 50$ mH, $L_1 = 10$ mH, $i = 5$ A:

$$T(\theta) = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} = \frac{25}{2}(-2L_1 \sin 2\theta) = -0.25 \sin 2\theta \text{ N·m}.$$

At $\theta = 30^\circ$: $T \approx -0.2165$ N·m.

A2. Synchronous power-angle $V_{LL} = 4.16$ kV, $E_{LL} = 3.6$ kV, $X_s = 1.2\,\Omega$ (per phase), $\delta = 25°$, $f = 60$ Hz, $p = 4$.

$$V_\phi = \frac{4.16}{\sqrt{3}} \text{ kV} \approx 2.402 \text{ kV}, \quad E_\phi \approx 2.078 \text{ kV}.$$

$$P_e = \frac{3V_\phi E_\phi}{X_s} \sin\delta \approx 5.28 \text{ MW}.$$

$$n_s = 1800 \text{ rpm}, \quad \omega_m = 188.5 \text{ rad/s}, \quad T_e \approx \frac{5.28 \times 10^6}{188.5} \approx 28 \times 10^3 \text{ N·m}.$$

6 Conclusions

RMF creates motion, flux paths decide how much magnetic coupling you get, and force/torque laws (Lorentz or co-energy) close the energy-conversion loop. Master these, and every machine model becomes transparent.