Time-Domain Circuit Analysis: A Practical Deep Dive

1 Why time-domain analysis still matters

Frequency-domain tools (Laplace, phasors) are fantastic, but when you're dealing with **transients**—power-up, switching events, ESD pulses, or any piecewise input—**time-domain circuit analysis** is the most direct way to see what really happens as a function of time. You get the actual waveforms: who overshoots, who rings, and how fast things settle.

2 The building blocks

Capacitor:

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

Inductor:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

Resistor:

$$v_R(t) = i_R(t)R$$

3 Natural vs. forced response

Total response = natural (homogeneous) + forced (particular), with constants fixed by initial conditions.

4 First-order time constants

For first-order circuits:

$$\tau_{RC} = R_{\text{th}}C, \qquad \tau_{RL} = \frac{L}{R_{\text{th}}}$$

A typical state variable:

$$x(t) = x(\infty) + \big(x(0^+) - x(\infty)\big)e^{-t/\tau}$$

5 Second-order RLC

For a series RLC:

$$\omega_n = \frac{1}{\sqrt{LC}}, \qquad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}, \qquad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

Underdamped ($\zeta < 1$), critically damped ($\zeta = 1$), overdamped ($\zeta > 1$).

6 Arbitrary inputs and convolution

$$y(t) = \int_0^t h(t-\tau)x(\tau)\,d\tau$$

7 Worked Examples

Example 1 (Beginner) — RC step

Given $R = 10\,\mathrm{k\Omega}$, $C = 10\,\mathrm{\mu F}$, $v_C(0^-) = 0$, step to $V = 5$ V at $t = 0$.

$$\tau = RC = 10{,}000 \times 10 \times 10^{-6} = 0.1 \text{ s}$$

$$v_C(t) = 5\left(1 - e^{-t/0.1}\right) \text{ V}, \quad t \geq 0$$

Example 2 (Intermediate) — RL decay

Given $L = 20\,\mathrm{mH}$, $R = 5\,\Omega$, $i_L(0^-) = 2$ A, no source after $t = 0$.

$$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{5} = 4 \text{ ms}$$

$$i_L(t) = 2e^{-t/0.004} \text{ A}$$

Find t when $i_L = 0.2$ A:

$$0.2 = 2e^{-t/0.004} \Rightarrow t \approx 9.21 \text{ ms}$$

Example 3 (Advanced) — Underdamped series RLC step

$R = 20\,\Omega$, $L = 10\,\mathrm{mH}$, $C = 10\,\mathrm{\mu F}$, $V_s = 10$ V applied at $t = 0$.

$$\alpha = \frac{R}{2L} = \frac{20}{2 \times 10 \times 10^{-3}} = 1000 \text{ s}^{-1}$$

$$\omega_n = \frac{1}{\sqrt{LC}} \approx 3162.3 \text{ rad/s}, \quad \zeta = \frac{\alpha}{\omega_n} \approx 0.316 < 1$$

$$\omega_d = \sqrt{\omega_n^2 - \alpha^2} \approx 2991.0 \text{ rad/s}$$

For a step in a series RLC initially at rest, the current is

$$i(t) = \frac{V_s}{L\omega_d}e^{-\alpha t}\sin(\omega_d t)$$

Numerically,

$$i(t) \approx 0.3345\,e^{-1000t}\sin(2991.0t) \text{ A}$$

8 Checklist

- Initial conditions are king: v_C and i_L are continuous through $t = 0$.
- Use R_{th} when computing τ.
- Classify 2nd order with ζ.
- "5 time constants" $\Rightarrow$ practically settled.

9 Common pitfalls

- Forgetting continuity of v_C and i_L.
- Miscomputing R_{th}.
- Wrong steady-state assumptions.
- Not re-evaluating initial conditions after multiple switch events.