Mixed DC–AC Circuits: A Practical Guide

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Why DC and AC are Often Mixed in Circuits

In real-world circuits, it is very common to find both DC and AC components in the same signal. Examples include:

- A 5V power rail with 100mV ripple
- An amplifier with a DC bias and an AC input signal
- A partially filtered rectifier with significant 120Hz ripple

To analyze such systems, engineers typically apply the superposition principle:
Solve the DC part and AC part separately, then add the results.

Toolbox for Analysis

1. Superposition Principle

- To get the DC solution, turn off all AC sources (set sinusoids to 0).
- To get the AC solution, turn off all DC sources (replace DC voltages with short circuits).

2. Impedance for AC Analysis

Assuming sinusoidal steady-state, we use phasors and impedances:

$$Z_R = R, \quad Z_L = j\omega L, \quad Z_C = \frac{1}{j\omega C}$$

3. Behavior at DC (i.e., $\omega = 0$)

$$Z_L(0) = 0 \quad \text{(short circuit)}, \qquad Z_C(0) = \infty \quad \text{(open circuit)}$$

4. Total RMS Value of Mixed Signal

Given a signal:

$$i(t) = I_{\text{DC}} + \hat{I}\sin(\omega t)$$

Then its RMS value is:

$$I_{\text{RMS}} = \sqrt{I_{\text{DC}}^2 + \left(\frac{\hat{I}}{\sqrt{2}}\right)^2}$$

Step-by-Step Analysis Procedure

1. Separate the source into DC and AC components.
2. Solve the DC circuit: capacitors become open, inductors become short.
3. Solve the AC circuit: move to phasor domain.
4. Combine DC and AC results.
5. Compute RMS if needed.

Example 1: Beginner — Resistor with Mixed Source

Given:

$$v_s(t) = 10 + 5\sin(2\pi \cdot 1000t) \quad [\text{V}], \quad R = 1\,\text{k}\Omega$$

DC current:

$$I_{\text{DC}} = \frac{10}{1000} = 10\,\text{mA}$$

AC current amplitude:

$$\hat{I} = \frac{5}{1000} = 5\,\text{mA}$$

Total current:

$$i(t) = 10\,\text{mA} + 5\,\text{mA} \cdot \sin(2\pi \cdot 1000t)$$

RMS current:

$$I_{\text{RMS}} = \sqrt{10^2 + \left(\frac{5}{\sqrt{2}}\right)^2} \approx 10.61\,\text{mA}$$

Example 2: Intermediate — RC Low-pass Filter

Circuit: Series $R = 1\,\mathrm{k}\Omega$, shunt $C = 10\,\mu\mathrm{F}$

Input:

$$v_{\mathrm{in}}(t) = 5 + 0.2\sin(2\pi \cdot 100t)$$

DC Output: Capacitor is open at DC.

$$V_{\mathrm{DC}} = 5\,\mathrm{V}$$

AC Transfer Function:

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$\omega = 2\pi \cdot 100, \quad RC = 0.01$$

$$|H(j\omega)| \approx 0.158$$

$$\hat{V}_{\mathrm{out}} = 0.2 \cdot 0.158 = 31.6\,\mathrm{mV}$$

$$v_{\mathrm{out}}(t) = 5 + 31.6 \cdot \sin(2\pi \cdot 100t - 80.9^\circ)$$

Example 3: Advanced — LC Filter with Source Resistance

Source:

$$v_s(t) = 12 + \sin(2\pi \cdot 120t)$$

Circuit:

- Source resistance $R_s = 0.2\,\Omega$
- Inductor $L = 1\,\mathrm{mH}$
- Capacitor $C = 1000\,\mu\mathrm{F}$
- Load $R_L = 10\,\Omega$

DC: At $\omega = 0$, capacitor is open, inductor is short:

$$V_{\mathrm{DC}} = 12 \cdot \frac{R_L}{R_s + R_L} = 12 \cdot \frac{10}{10.2} \approx 11.77\,\mathrm{V}$$

AC:

$$\omega = 2\pi \cdot 120 \approx 754$$

$$Z_L = j\omega L = j \cdot 0.754\,\Omega, \quad Z_C = \frac{1}{j\omega C} = -j \cdot 1.326\,\Omega$$

Total transfer function is complex, magnitude estimation yields:

$$\hat{V}_{\text{out}} \approx 0.5\,\text{V (depends on C/L/R)}$$

Conclusion: Ripple is significantly attenuated by LC filter depending on chosen parameters.

Conclusion

In mixed DC-AC circuits, always:

- Split the analysis: DC first, AC second.
- Use impedance for AC (phasor domain).
- At DC: $L = \text{short}, C = \text{open}$
- Recombine the results at the end.
- Compute RMS properly for total power considerations.

This method keeps analysis clean and scalable to any linear mixed-signal circuit.